XXV SIMPÓSIO BRASILEIRO DE RECURSOS HIDRÍCOS

INTERWAVE ANALYZER - INVESTIGATING THE HYDRODYNAMIC OF LAKES FROM TEMPERATURE DATA

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Abstract: Biogeochemical fluxes and water masses transport in thermally stratified lakes are often dominated by internal waves. These phenomena are triggered by baroclinic forces generated by the interaction of wind-driven flows and temperature variation along the water column. Although this mechanism is well documented, the complexity of the mathematical tools and concepts engaged in examination of internal waves can make it difficult to establish direct links between internal wave dynamics and biogeochemical cycles or biological processes. This study presents an application of an open-source software developed for the analysis of internal waves and their influence on the hydrodynamics of lakes and reservoirs. Various mathematical tools are combined to improve understanding of how lake ecosystem might be affected by internal waves. The application is carried out with temperature and meteorological data from Lake Milada (Czech Republic), where internal waves have already been detected and the influence of internal seiche on fish distribution has been demonstrated.

Keywords: Internal seiche, Lake mixing, Lake Milada

INTRODUCTION

The hydrodynamics of lakes and reservoirs are strongly dominated by baroclinic forces (Ahmed et al., 2014; Hingsamer et al., 2014), which can induce currents that are not necessarily aligned with the wind direction. Insufficient understanding of the hydrodynamics of lakes and reservoirs can result in potential misinterpretation of biogeochemical fluxes due to the complicated velocity field generated by baroclinic forces. Among the most important phenomena affecting the hydrodynamics of density-stratified water bodies, such as lakes and reservoirs, are internal waves (Mortimer, 1952). Wind-induced internal waves at the basin scale (Figure 1) are stationary internal waves that are triggered by the wind blowing across the lake. These waves are often observed when the force equilibrium between wind shear and horizontal pressure (Wedderburn number) is of the order of one (Antenucci et al., 2000; Roberts et al., 2021; Valbuena et al., 2022).

This type of wave occurs inside the lake and can cause strong currents that can affect biogeochemical cycles and physical properties. Internal waves can alter a variety of properties of the lake environment including an increase in turbulence (Remo Cossu & Wells, 2013; de Carvalho Bueno et al., 2023), a change in light availability (Hingsamer et al., 2014), an induction of sediment

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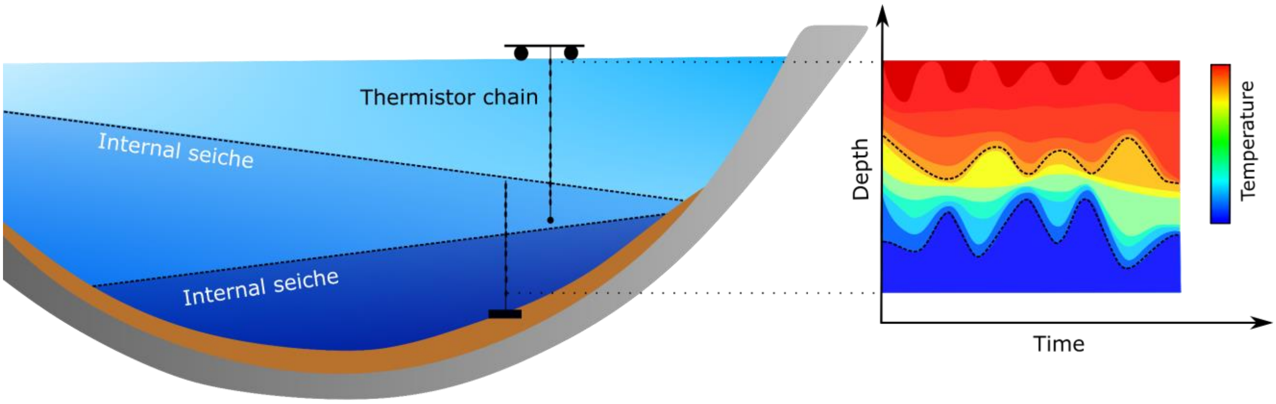
transport (Valipour et al., 2017) leading to a change in the biogeochemical cycles (Pannard et al., 2011), a change in the spatial concentration of dissolved oxygen (Valerio et al., 2019), and the vertical distribution of fish (Jarić et al., 2022). Moreover, the mixing caused by internal waves can promote the growth of phytoplankton (Pannard et al., 2011), which forms the basis of the lake's food chain, contributing to the change in water quality of these ecosystems.

Figure 1. Schematic overview of the occurrence of an internal seiche of the second vertical mode. Different shades of blue indicate layers with different temperatures. Brown and silver indicate the sediment and the solid boundary of the lake, respectively. Internal seiches occur at temperature interfaces and are often monitored by the thermistor chain, which records the temperature profile over time (right figure). The dashed lines indicate the interfaces where the internal seiches predominate. The rapid temperature oscillation in the surface region (dark red) shows the diurnal temperature variation caused by diel changes of solar radiation.



Unlike surface waves, internal waves in thermally stratified lakes are not easily detected because they require measurements of temperature or current velocity profiles. To characterize these waves, limnologists often use a variety of methods, including spectral analysis, numerical models based on governing equation of motion, and densimetric dimensionless numbers (de Carvalho Bueno, Bleninger, et al., 2020). A comprehensive study of internal waves requires expertise in fluid dynamics, which is a challenge for those who lack knowledge in this field, but are interested in understanding the link between lake hydrodynamics and biogeochemical cycles or biological interactions.

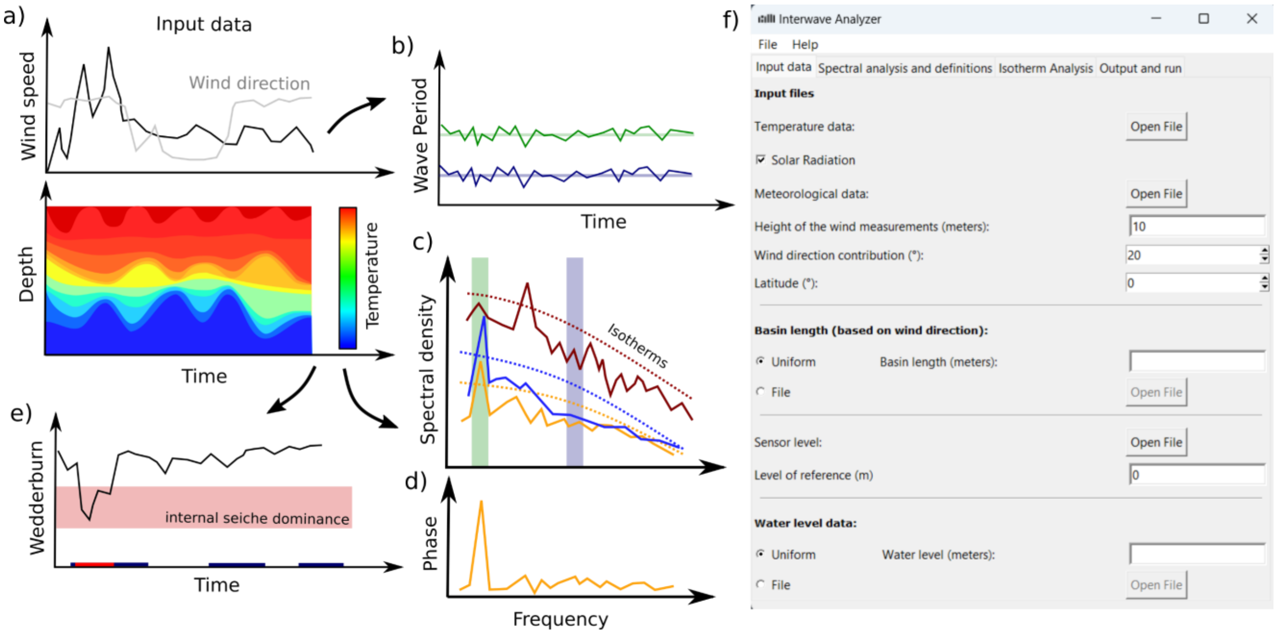
The aim of this paper is to demonstrate an application of a powerful open-source tool Interwave Analyzer (de Carvalho Bueno, Bleninger, et al., 2020) to detect and characterize the dynamics of internal waves in thermally stratified lakes. The software uses multilayer models to calculate lake mixing parameters, stratification indices, wave characteristics, and spectral analyses of temperature and meteorological data. To demonstrate the power of the tool, we used data from Lake Milada, where the seiche has been already studied (Jarić et al., 2022). The Interwave Analyzer facilitates interdisciplinary research by examining the links between internal wave dynamics and biogeochemical cycling in aquatic ecosystems and improving access to state-of-the-art theoretical and empirical knowledge in physical limnology.

METHODS

The Interwave Analyzer is open-source software designed to provide detailed characterization of the internal waves dynamics and water mixing regimes in thermally stratified lakes and reservoirs. These parameters are calculated using water temperature and wind velocity measurements (de Carvalho Bueno, Bleninger, et al., 2020). Unlike the Lake Analyzer (Read et al., 2011), the Interwave Analyzer can reveal many characteristics of the internal seiche and lake mixing regimes, including internal waves of higher vertical and horizontal modes, wind-wave resonance, internal wave amplitudes, internal seiche degeneration, and lake mixing regimes.

The Interwave Analyzer is coupled with a simplified numerical model, which can be used to estimate the wave period of various internal seiche modes (Figure 2a). To reveal the oscillation pattern in the isotherms and meteorological data, the software also performs spectral analysis based on Fourier and wavelet analyses. The theoretical periods of the different modes of internal seiches are plotted along with the spectra to facilitate the identification and characterization of internal seiches (Figure 2c). To identify internal seiches with higher vertical mode, a phase analysis is performed in different isotherms to reveal the out-of-phase response (Figure 2d).

Figure 2. The graphical user interface (GUI) of Interwave Analyzer and a simple overview of some input data and output analyses performed by Interwave Analyzer. a) The two main input data, the time series of speed and direction of wind and a lake water temperature profile. The arrows show some output results from the input data. b) Time series of the theoretical wave period for different modes (different colors). b) Fourier analysis of different isotherms (solid colored lines) with the mean red noise spectrum for each time series at a 95% confidence level (dashed lines). The green and blue shaded vertical regions shows the mean period of each mode estimated in Figure 2b. d) Phase analysis performed between two different isotherms to highlight out-of-phase response along the frequency range, and e) Wedderburn number time series. The red-shaded region indicates the period of internal seiche dominance according to the lake mixing regimes proposed by Spigel and Imberger (1980). f) Input data tab, where some general parameters related to the analyzed lake/reservoir must be specified (e.g., temperature and meteorological data, lake latitude, and basin length).



☑ Solar Radiation

Height of the wind measurements (meters): 10

Wind direction contribution (°): 20

Latitude (°): 0

● Uniform Basin length (meters):

○ File

Level of reference (m) 0

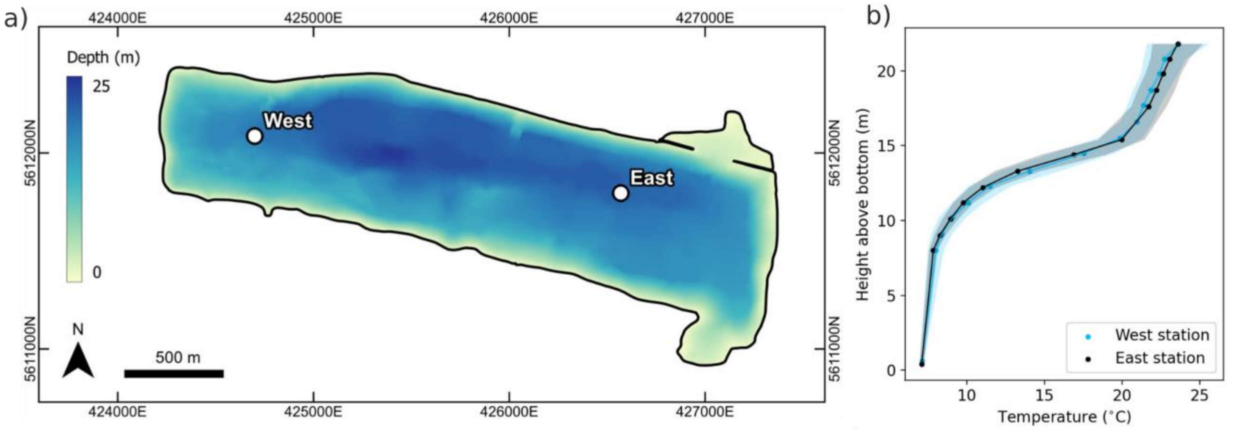
● Uniform Water level (meters):

○ File

The software also detects, based on the Weederburn number (Imberger & Patterson, 1989), periods of potential internal seiche activity (Figure 2e), mixing regime of the lake and how the internal seiche may degraded during the analyzed period. It helps characterize the mixing of the lake and indicate how the internal seiche may contribute to the transport and mixing of water masses.

To demonstrate the performance of the software, we used temperature and meteorological data from Lake Milada (Figure 3), where internal seiches were revealed and the effect of seiche dynamics on four fish species was studied (Jarić et al., 2022). Lake Milada (50°39'N, 13°58'E) is a small, shallow and man-made lake located in the Czech Republic (Figure 3). The lake is classified as oligotrophic and has a surface area of 2.5 km^2 and a maximum depth of 25 m. It has a simple, elongated shape, with a flat bottom profile and an open landscape without forest cover or other structures that could interrupt or attenuate wind flow. Prevailing winds tend to follow the direction that closely coincides with the longer axis of the lake, allowing frequent occurrence of internal waves (Jarić et al., 2022). The lake was studied from April 2015 to March 2016 (Jarić et al., 2022), combining hydrodynamic, meteorological and fish movement monitoring. In this study, only the July 2015 meteorological and hydrodynamic data were analyzed (fish movements were not analyzed here because they were beyond the scope of the study). Vertical temperature profiles were measured using two thermistor chains equipped with 15 temperature loggers (HOBO Pendant temperature/light 64 K, Onset, USA, resolution 0.14°C, accuracy ±0.53°C) deployed 2 km apart in the eastern and western parts of the lake with a sampling interval of 5 minutes (Figure 3). Wind direction and wind speed were measured by the weather station in the center of the lake (Figure 3a) at 30 minutes intervals.

Figure 3. Lake Milada and thermistors chains. a) Bathymetric map of Lake Milada, with points to the west and east indicating the locations of thermistor chains used to record water temperature. b) Mean vertical temperature profiles measured at both stations, with standard deviations indicated by the black (east) and blue (west) shaded areas. Dot markers indicate the position of the thermistors along the water column.



Although the software requires a number of input parameters (e.g., basin length, cut-off frequencies for filtered spectral analysis, and metalimnion threshold) most are single values used to characterize the lake or calibrate the processing steps. The most important input parameters are the time series of the temperature profiles (.tem) and the wind velocity and direction (.win). In the presented dataset we assumed a fixed basin length of 3 km, and neglected the variation of wind fetch as a function of wind direction. All data, including data file paths and calibration parameters, must be specified by the user in the graphical user interface (Figure 2f). The software was run separately for the West and East stations. All results presented in this study are based solely on the output data from the Interwave Analyzer.

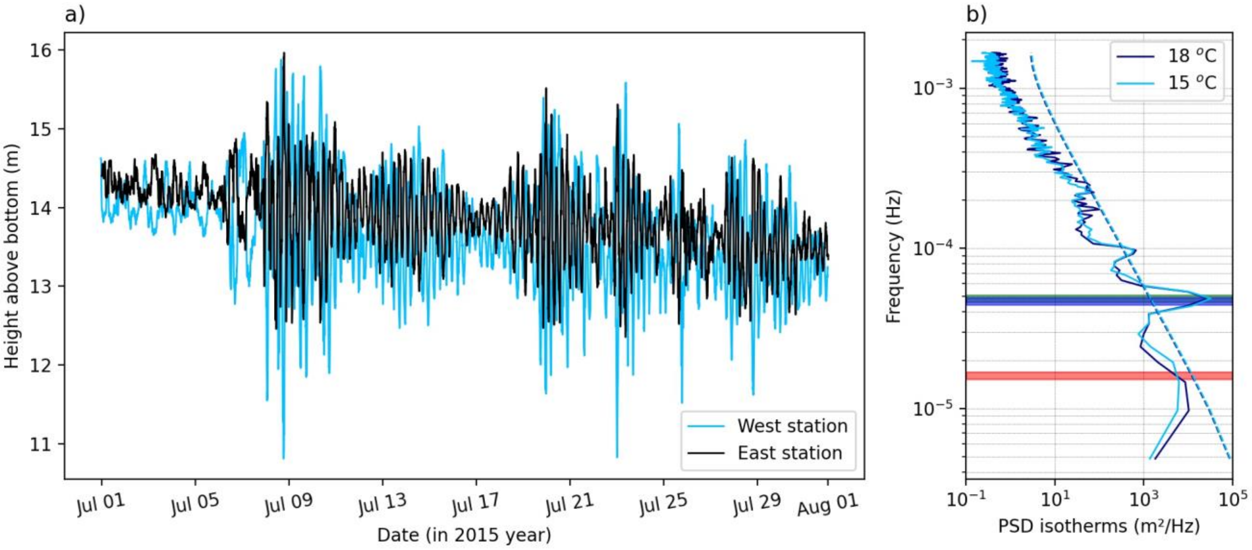
All input data used to run Interwave Analyzer and all results, are available in the repository https://github.com/buenorc/interwave_milada.git. The source code for the software can be downloaded from the Interwave Analyzer repository: https://github.com/buenorc/interwaveanalyzer. The code can be run in any Python interpreter (e.g., Spyder) and does not require any knowledge of the programming language, as the user can work directly in the friendly graphical user interface. The software also has a simple executable version that can be requested through this link: https://sites.google.com/view/interwaveanalyzer. Version 1.01.1 was used in this study. For more information on downloading and running Interwave Analyzer, see the user (de Carvalho Bueno, Lorke, et al., 2020).

RESULTS AND DISCUSSION

Isotherms go beyond their static representation, and serve as dynamic interfaces where internal waves can generate and propagate. This phenomenon reveals the underlying complexity of natural systems, and provides insight into the fundamental dynamics of temperature distribution and its influence on physical processes in thermally stratified lakes. The Interwave Analyzer can be used to determine different isotherms depending on user specifications. In this study we selected 4 different isotherms (20 ºC, 18 ºC, 15 ºC, and 13 ºC), covering the whole range of temperature observed in the analyzed period. Time series of 15 ^{o}C isotherm from the west and east stations of Lake Milada show strong fluctuations during the studied period (Figure 4a). The characteristic of the oscillation patterns of the isotherms can be revealed by spectral analysis, based on Fourier (Vidal et al., 2013) and Wavelet transforms (Stevens, 1999).

The power spectral densities (based on the Fourier transform) of the isotherms obtained by the Interwave Analyzer reveal two significant peaks at 4.5×10^{-5} Hz (5 h 43 min) and 9.5×10^{-5} Hz (2 h 50 min) at both stations (only the west station is shown in Figure 4b). The highest spectral energy is observed at the 15 °C isotherm (Figure 4b), which is consistent with the mean thermocline depth (Figure 3b).

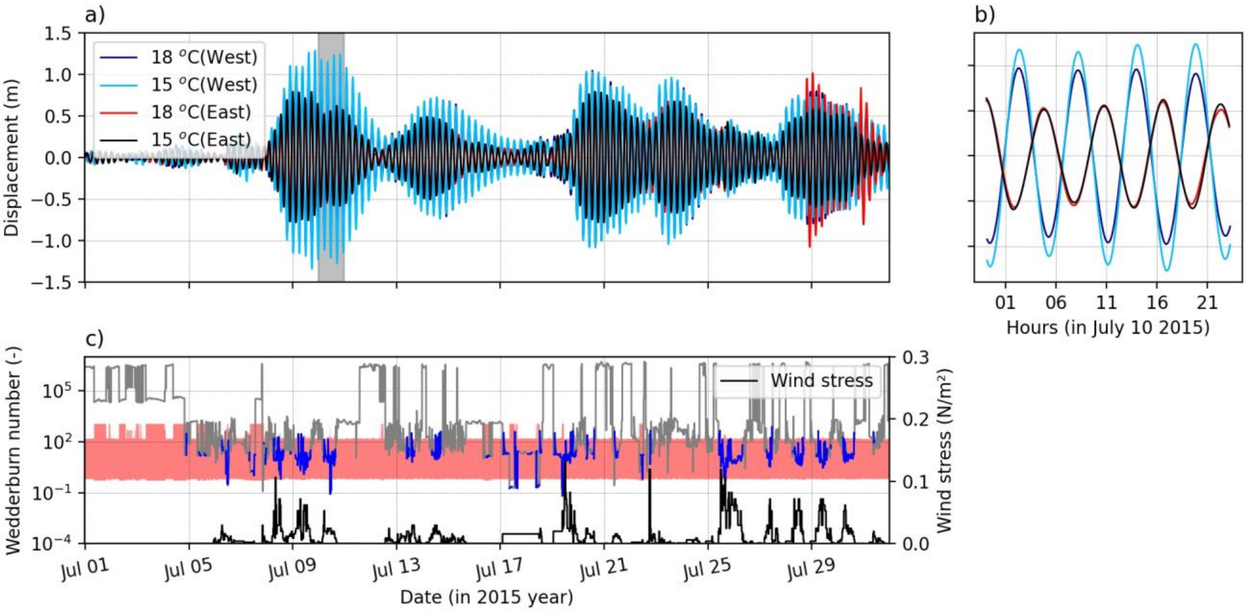
Figure 4. Analysis of the time series of isothermal depths at the sampling stations. a) Time series of the 15 $^{\circ}$C isotherm at the west (blue line) and east (black line) stations. b) Power spectral density (PSD) of isotherm displacements for the 15 ^{o}C and 18 ^{o}C isotherms at the west station. The vertical green, blue, and red bars in the spectrum mark the frequencies of internal seiches estimated by the multilayer model for the first, second, and third vertical modes, respectively. The width of the colored boxes indicates the variability of the estimated periods of internal waves based on 20% variations of the specified lake length (L=3 km). The dashed lines indicate the mean red noise spectrum for the time series at a 95% confidence level.



The lowest frequency peak (4.5×10^{-5} Hz) is consistent with the theoretical period of the first (5 h 44 min ± 17 min, Figure 4b; green shaded box) and second internal seiche modes (5 h 58 min ± 18 min, Figure 4b; blue shaded box) estimated by the multilayer model (Münnich et al., 1992) embedded with the Interwave Analyzer. Although this suggests that the highest energetic peak might be associated with oscillations of the first or second vertical mode, the phase-shift analysis performed with the Interwave Analyzer showed no phase-shifted responses in different isothermal pairs. The time series of the isothermal depth bandpass filtered at cutoff frequencies of the theoretical internal wave frequency (corresponding to a period of 5 h 44 min ± 17 min) shows that the two selected isotherms (18 ^{o}C and 15 ^{o}C) fluctuate in phase throughout the studied (Figure 5), indicating that internal seiche of second vertical mode is not generated during the analyzed period. Another Interwave Analyzer result that supports this observation is the spectral analysis performed along the water column (see Interwave Analyzer output results from Lake Milada available at https://github.com/buenorc/interwave_milada.git). Temperature fluctuations at 4.5 10^{-5} Hz (5 h 43 min) are observed as a single peak at mid depth, near the thermocline. The occurrence of an internal wave of second vertical mode would produce at least two distinct peaks along the water column, probably at 11 and 17 meters above the bottom, as predicted by the Interwave Analyzer considering a three-layer system.

Analysis of the isothermal depth time series with bandpass filtering reveals five periods of significant fundamental internal seiche activity (Figure 5a). The strongest wave, which occurred at the end of July 8, generated a vertical displacement of about 2.6 m (Figure 5a), followed by four other events with magnitudes between 1.5 and 2.0 m according to west station data. The observed mean vertical displacement is 160 % higher than the vertical displacement at the east station, which could be due to the different distances between the two stations and the wave node. The east station is located in a deeper area, probably closer to the wave node than the west station, so a lower vertical displacement could be expected.

Figure 5. The occurrence and potential periods indicated by dimensionless physical parameters. a) Time series of the isothermal depth bandpass filtered with cutoff frequencies of 4.9 10^{-5} and 5.1 10^{-5} Hz (theoretical internal seiche frequency of the first vertical mode). The vertical gray bar indicates the period of isothermal fluctuations highlighted in Figure 5b. c) Time series of the Wedderburn number and the wind shear stress. The blue color indicates the period of potential internal seiche activity estimated by the Interwave Analyzer based on constant wind events that can generate seiche. The software only considers wind events longer than ¼ of the theoretical period of the internal seiche and act in a constant direction. The black line is the wind shear stress.



Water temperature is a critical environmental factor for aquatic ectotherms, and a vertical shift in thermocline of 1 to 3 meters (12% of total water depth) in a shallow Milada Lake can have several biological consequences. Jarić et al. (2022) studied the vertical movement response of four fish species to seiche in the lake. They showed that all species studied preferentially occupied the epilimnion in summer and changed their depth rapidly to avoid the cold, hypolimnetic, upwelling water during seiche events. Other studies have also found effects of seiches on the vertical distribution of fish (Aspillaga et al., 2017; Brooks et al., 2022) or zooplankton (Easton & Gophen, 2003) in various aquatic ecosystems. This suggests that thermocline shift during seiche events may result in a temporary reduction in habitat for species responding to a seiche, which in turn may have implications for predation and other inter- and intraspecific biological interactions, and ecosystem dynamics. In addition, climate change is expected to affect physical characteristics, including water temperature and hydrology of lakes (O'Reilly et al., 2003, 2015), and potentially affect internal seiche dynamics (Cossu et al., 2017). However, the effects of seiches on aquatic organisms have been poorly studied so far, due to the complexity of detecting and describing seiche dynamics. We believe that the Interwave Analyzer can greatly facilitate the seiche data processing and promote further research on ecosystem functioning and response to internal seiche.

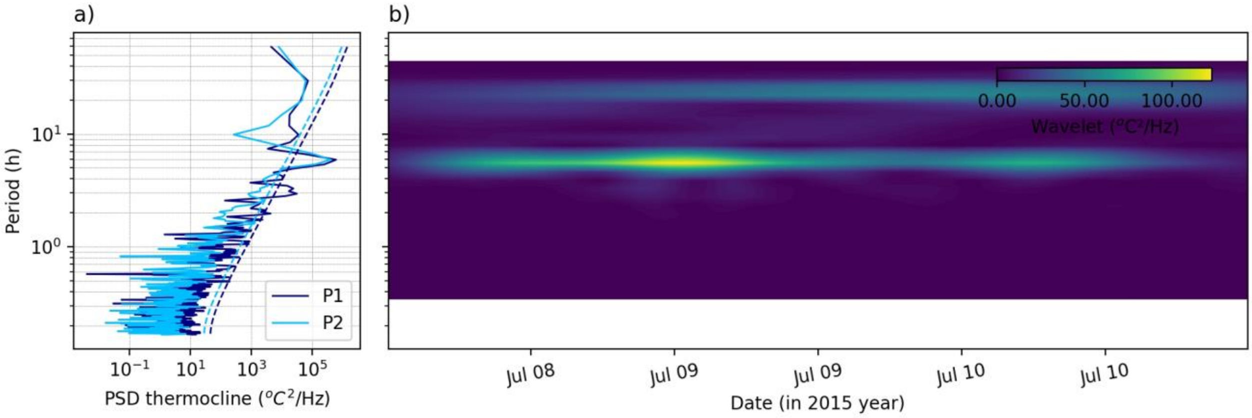
The results of the Interwave Analyzer suggest that fundamental internal seiches detected in Lake Milada are triggered by strong wind events, as observed by the increase in wind shear stress (Figure 5c) during periods of strong isothermal fluctuations (Figure 5a). Internal waves with amplitude greater than 50 cm are often observed when the wind shear stress exceeds 0.08 N m^{-2}, except for oscillations triggered in early July 14, which were generated by much lower wind shear stress (0.02 N m^{-2}). Although wind speed decreased during this period, the pressure gradient created by stratification also decreased, lowering the Wedderburn number (Figure 5c). The Interwave Analyzer not only shows the period of potential internal seiche activity based on the standard Wedderburn number (gray line; Figure 5c), but also filters the Wedderburn number (W) considering only constant wind events that can generate internal seiche (blue markers; Figure 5c).

The analysis predicted that buoyancy dominates all processes and internal waves are not dominant ($W > 100$) during the first 6 days of the analysis. The prediction is consistent with the internal seiche activity estimated using the isothermal depth band pass filter (Figure 5a), which shows a minimal fluctuation associated with internal seiche activity. After the first six days, internal seiche determined from the 15°C isotherm dominates (Figure 5a), followed by low Wedderburn number events (Figure 5c). This analysis is consistent with previous observations in Lake Milada (Jarić et al., 2022), where high amplitude waves were detected during the same analysis period.

The second peak (2 h 50 min) identified from the power spectral densities of 15 ^{o}C isotherms (Figure 4b) has significantly lower energy than the lowest-frequency peak discussed previously, and has maximum amplitude of 50 cm. Observations in the southern Aral Sea have attributed high-frequency peaks to transverse internal seiches induced by winds blowing perpendicular to the main direction of the lake (Roget et al., 2017). Based on a multilayer model, the Interwave Analyzer reveals that this oscillatory pattern is unlikely to be caused by an internal transverse wave, as modeling results show that the wave period is much higher than the theoretical period estimated by the model (1h 20 min).

The wavelet analysis reveals that the peak energy of the high-frequency internal wave coincides with the time of maximum spectral energy corresponding to the fundamental internal seiche identified with a period of 5 h 43 min (Figure 6). This could indicate that the high-frequency waves are induced by internal seiche degeneration. Internal waves with high amplitude at the basin scale become steep faster and transfer energy to smaller scales of motion. Due to nonlinearity, the internal wave at the basin scale transforms into a sequence of individual internal waves (Lorke, 2007). This degeneration process is classified based on the dimensionless Wedderburn number and the time scales associated with the internal seiche condition (Horn et al., 2001).

Figure 6. Spectral analysis of the 15 ˚C isotherm at the west station for two subperiods. a) Power spectral density of the 15 ˚C isotherm for the period from 08 to 10 July (P1; dark blue solid line) and from 13 to 15 July (P2; light blue solid line). The dashed lines show the mean red noise spectrum for the time series at a 95% confidence level. b) Wavelet analysis of the 15 ˚C isotherm for period P1.



According to the results of the Interwave Analyzer, the period of high-amplitude internal seiches falls into the regime of solitons (Horn et al., 2001), in which fundamental internal seiches can degenerate into high-frequency internal waves. Comparing two periods with different internal seiche amplitudes, July 08 - 10 (P1) and July 13 - 15 (P2), we observed a higher spectral energy associated with high-frequency oscillatory motions (2 h 50 min) during period P1 than during period P2. During period P1, the internal waves at the basin-scale had a higher amplitude and a lower Wedderburn number (W=6), suggesting a process of degeneration in which the basin-scale internal wave transformed into a train of high-frequency internal waves. In period P2, Wedderburn number was slightly higher than in period P1 (W=10). It indicates that the wave transited to a damped basin-scale wave, when the wave is damped by viscosity before steepening and transitioning to high-frequency internal waves.

CONCLUSION

The Interwave Analyzer facilitates the analysis of lake and reservoir hydrodynamics using temperature data. It provides an in-depth analysis of internal waves, giving information on the mixing regimes in lakes and internal wave field characteristics, including wind wave resonance, internal wave amplitudes, frequencies, and modes, and type of internal seiche degeneration. The application of Interwave Analyzer to Lake Milada data demonstrated the power of the software to provide insights into the prevalence of internal waves, their generation and decay. It further showed that the software can provide more information to facilitate interdisciplinary research to draw connections between internal wave dynamics and biogeochemical cycles or biotic interactions, promoting the integration of state-of-the-art physical limnology into broader environmental assessments.

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